





$$\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_3}{\partial x_3 \partial x_2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_2^2}$$

## Cocoa based Agroforestry as a Complex System



Source: Jagoret et al. 2017. Agron.

Provide many ecosystem services and environmental benefits: stable production, sustainable cropping system, food security, ....

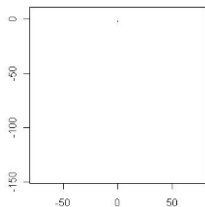
Complex systems with many interactions between trees and the environment (climate, soil, socio-economic context, ...)

- → **Develop models to improve understanding of systems** and thus able to design productive and sustainable cropping systems



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## Continuous mathematical models for plant growth



Maize root growth simulations

**Phase I** :  $t < 48d$

Data: Loïc Pagès (PSH, Avignon)



(a) Apex density (from the data)



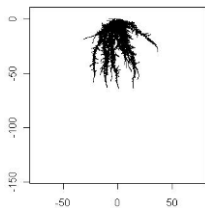
(b) from C-Root

Bonneu, PhD thesis, 2011



$$\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_3 \partial x_2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_2^2}$$

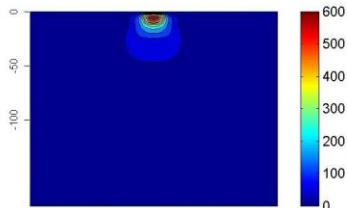
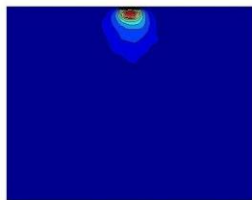
## Continuous mathematical models for plant growth



Maize root growth simulations

**Phase II** :  $t \geq 48d$

Data: Loïc Pagès (PSH, Avignon)



(a) Apex density (from the data)

(b) from C-Root

Bonneu, PhD thesis, 2011



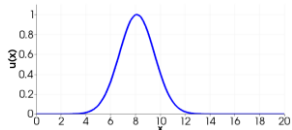
$$\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_3 \partial x_2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_2^2}$$

## The C-Root model: a continuous mathematical model

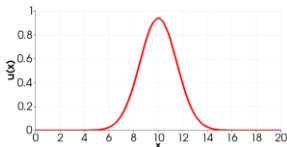
Link root growth processes with usual physical operators

$$\frac{\partial \mathbf{u}}{\partial t} = - \mathbf{v} \cdot \nabla \mathbf{u} + \nabla \cdot \mathbf{D} \nabla \mathbf{u} + \mathbf{R} \mathbf{u}$$

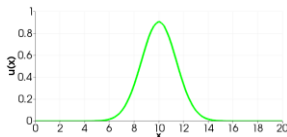
*advection*
*diffusion*
*reaction*



transport of  $u$  by the vector  $\mathbf{v}$



spreading out of  $u$  in all directions when  $\mathbf{D}$  is a diagonal matrix



creation or destruction of  $u$  according to the sign of  $\mathcal{R}$

$$\mathcal{R} = \beta - \mu$$

(branching and mortality)

Bonneu, PhD thesis, 2011

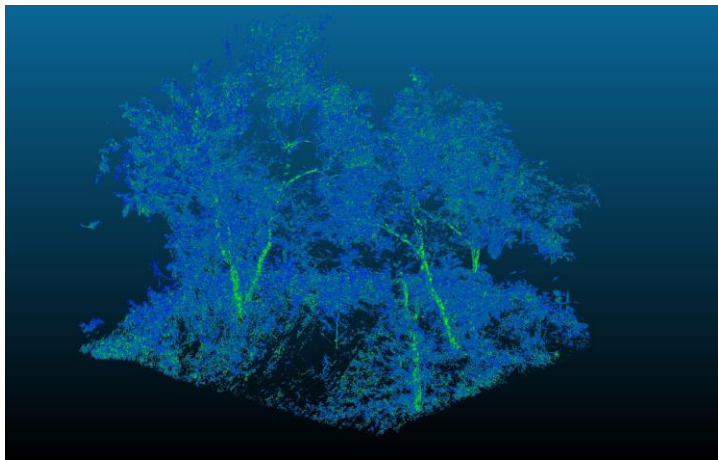
Bonneu, Dumont, Rey, Jourdan, Fourcaud, Plant and Soil 354, 2012



$$\frac{\partial x_1}{\partial x_1 \partial x_2} + \frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial x_2 \partial x_1}{\partial x_2 \partial x_3} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial x_3 \partial x_1}{\partial x_3 \partial x_2} + \frac{\partial^2 u_3}{\partial x_3^2} + \frac{\partial x_1}{\partial x_1^2} + \frac{\partial x_2}{\partial x_2^2} + \frac{\partial x_3}{\partial x_3^2} + \frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_3}{\partial x_3^2}$$

## Terrestrial lidar for data collection

Tool used to assess the dynamics of leaves and to estimate wood biomass

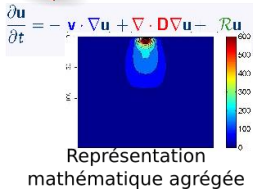
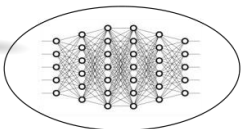
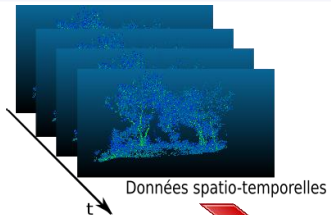


Cocoa trees in Bokito



$$\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_3}{\partial x_3 \partial x_2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_2^2}$$

# The idea of the project: link between models and data





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## How to identify PDE thanks to neural networks ?

Papers	Neural network architectures	Real data ?
J. Berg, K. Nystrom, 2019	FFNN	yes
Z. Long et al, 2018	CNN, FFNN	no
M. Raissi et al. 2017	FFNN	no

Challenge: most of existing work are based on synthetic data in physics and not on field data





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## Overview of the Deep2PDE project

### Goal

uncover a PDE model from plant growth data using neural networks

### Tasks

On field data collection and synthetic data

Design and training of a neural network



Kick-off workshop of the project in Yaoundé in March 2019



$$\frac{\partial x_1}{\partial x_1 \partial x_2} + \frac{\partial^2 u_1}{\partial x_2^2} + \frac{\partial x_2 \partial x_1}{\partial x_3 \partial x_2} + \frac{\partial^2 u_3}{\partial x_3^2} + \frac{\partial x_1}{\partial x_2^2} + \frac{\partial x_2}{\partial x_3^2} + \frac{\partial^2 u_2}{\partial x_3^2} + \frac{\partial^2 u_2}{\partial x_3^2} + \frac{\partial^2 u_2}{\partial x_3^2}$$

## Simplified data generated by AmapSim

AmapSim: a model and software for architectural plant growth and development simulations

Design from botanical knowledge of plants

- Use this software to generate "not-too-synthetic" data



